Fundamental approach to failure statistics of optical glass fibres

J. E. RITTER, JR, K. JAKUS
Mechanical Engineering Department, University of Massachusetts, Amherst, Massachusetts 01003, USA

Tensile strength data of fused silica optical fibres was analysed without making any *a priori* assumptions regarding the flaw density distribution. Based on the strength data of short length specimens (0.05 to 0.6 m), predictions of the strengths of long length specimens (500 to 1100 m) were in good agreement with actual data. The advantages of a fundamental approach to the statistical analysis of failure of optical glass fibres compared with the more widely used Weibull statistics are discussed.

1. Introduction

The statistical analysis of strength data to evaluate the failure probability associated with the use of optical glass fibres is an important problem. The problem essentially consists of two parts: the acquisition of reliable strength data and the development of a statistical model to describe failure. Ideally, the appropriate statistical analysis will permit the strength distribution function to be well defined from a relatively small number of tests. This distribution function can then be used to predict effects of size, stress state, etc. on the failure probability at various stress levels.

The statistical analysis of fracture strength of optical glass fibres has generally followed the approach pioneered by Weibull [1–3] who showed that if a specimen of surface area A contains a statistical distribution of noninteracting flaws, the probability of failure (ϕ) is determined from

$$1-\phi(S) = \exp\left[-\int_A \mathrm{d}A \int_0^S g(S)\mathrm{d}S\right] \quad (1)$$

where $g(S)\mathrm{d}S$ is the number of flaws per unit area with a strength between S and $S+\mathrm{d}S$. Weibull then assumed an asymptotic functional form for $g(S)$

$$\int_0^S g(S)\mathrm{d}S = \left(\frac{S-S_u}{S_0}\right)^m \quad (2)$$

where S_u is the lower limit on strength, S_0 is the scale parameter and m is the shape parameter. By assuming this function, Equation 1 can be integrated and the distribution parameters S_u, S_0 and m can be deduced from strength failure probability data.

Weibull's analysis has been extended to include the case where the strength of optical fibres is controlled by a bimodal flaw distribution [4–9]. In this case the strength data are fitted to two distributions of the form of Equation 2. Unfortunately, real flaw distributions on optical glass fibres are not necessarily best characterized by Equation 2 and a more fundamental approach to statistical analysis is preferred. Such an approach has recently been developed [10, 11] in which it was shown that by introducing the sample stress distribution, Equation 1 can be manipulated to obtain $g(S)$ in terms of failure probability without assuming *a priori* that $g(S)$ has a specific functional form. These solutions are quite straightforward for the uniaxial tensile test which is widely used for obtaining strength data for optical glass fibres. The purpose of this paper is to apply this fundamental statistical approach to strength test data for optical glass fibres and to demonstrate its advantages over the more widely used Weibull statistics.

2. Data analysis

In a uniaxial tensile test the specimen fails at the maximum stress, S_m, and Equation 1 becomes

$$\xi(S_m) = -\ln[1-\phi(S_m)] = A\int_0^{S_m} g(S)\mathrm{d}S \quad (3)$$

0022–2461/81/071909–04$02.40/0

where A is the surface area under test and is equal to $2\pi rL$ with r being the fibre radius and L being the gauge length. Differentiating Equation 3 with respect to S_m gives

$$g(S_m) = \frac{1}{A}\frac{d\xi(S_m)}{dS_m}. \qquad (4)$$

Thus, for uniaxial tension the flaw density, $g(S_m)$, at any stress, S_{m1}, is proportional to the derivative of the $\xi(S_m)$ against S_m curve at S_{m1}. Once $g(S_m)$ is determined, it can then be used to predict failure strengths for long length fibres as well as fibres under different modes of loading. For example, to make failure predictions for long length fibres in uniaxial loading, $g(S_m)$ is simply integrated using Equation 3. For modes of loading other than uniaxial, where the stress distribution is more complicated, the integration equation becomes complex and the reader is referred to the paper by Evans and Jones [11] for example of cases where bending stresses are present.

Data analysis can be illustrated by examining data obtained for two groups (B and D) of optical glass fibres [4]. Both groups were drawn in an electric furnace and were polymer coated in line with the major difference being that Group D fibres were proof tested at 207 MPa. Group B fibres were tested with gauge lengths of 0.05 and 0.61 m and Group D fibres with a gauge length of 0.05 m. The numbers of samples tested in each data set was 270, 270 and 400, respectively.

The data analysis procedure first requires that the test data be ordered and then the cumulative failure probability $\phi(S_m)$ be determined as a function of the fracture stress, S_m. The quantity $\xi(S_m)$ $[\equiv -\ln[1-\phi(S_m)]]$ is then evaluated and plotted as a function of S_m, see Fig. 1. Superimposed in Fig. 1 is the cubic polynomial representation of the data. The fitting technique was designed to permit the curve to follow broad peaks in the data, while smoothing out localized oscillations. This was done because broad peaks often have a physical significance that relates to the flaw distribution but localized oscillations are generally a result of statistical variations that can cause gross fluctuations in the derivatives which impede the interpretation of the derived strength distributions. The derivatives of $\xi(S_m)$ that determine $g(S_m)$ can then be deduced directly from the polynomials. Fig. 2 comprises the flaw density curves for the 0.05 and 0.061 m Group B and 0.05 m Group D fibres that were derived from the data plots such as shown in Fig. 1.

3. Interpretation

The flaw density curves (Fig. 2) contain several implications concerning the nature of the strength controlling defects and the size dependence of fracture strength. Both of the flaw density plots for the Group B fibre samples show two distinct flaw populations. The low strength flaw popu-

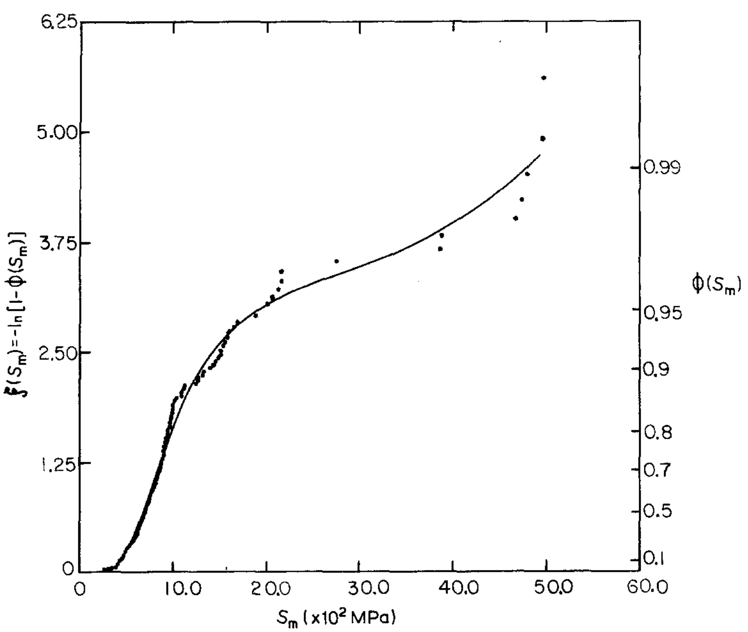
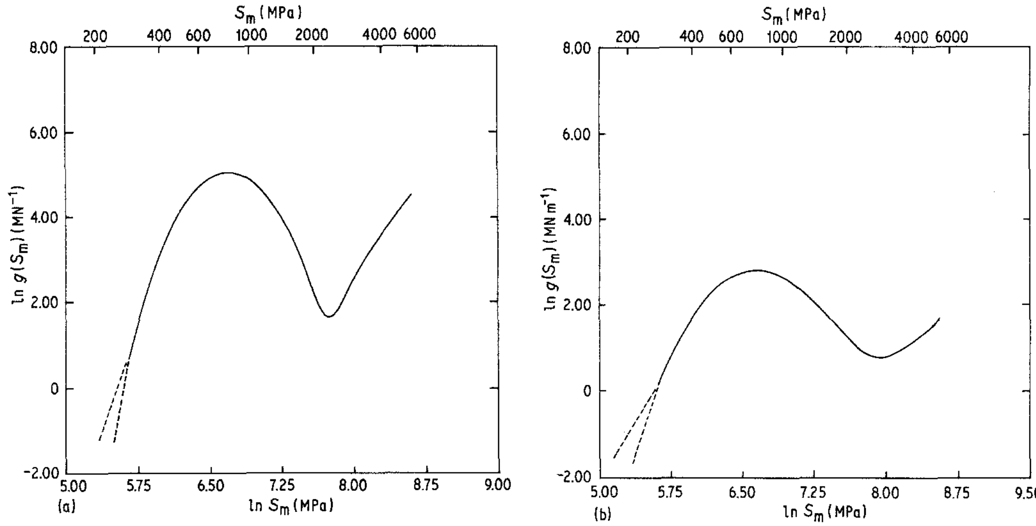


Figure 1 Typical plot of failure probability against fracture strength; data are for 0.61 m group B fibres. For clarity not all data points are shown at low failure probabilities.



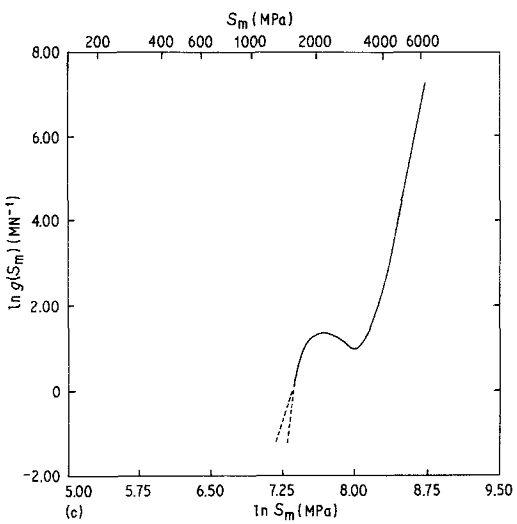


Figure 2 Strength distribution function, $g(S_m)$ against fracture strength for (a) 0.05 m group B fibres, (b) 0.61 m group B fibres , and (c) 0.05 m group D fibres.

lation is centred in each figure at about 850 MPa, while the high strength flaw population is continuously increasing up to the maximum strength of the data (about 5400 MPa). On the other hand, the flaw density plot for the Group D fibres shows no low strength flaw population but instead there appears to be two high strength flaw populations. Although the Group D fibres were proof tested, it is quite unlikely that this eliminated the low strength population exhibited by Group B fibres since this population is centred about 850 MPa, while proof testing was at 207 MPa. Thus, the reason for the lack of the low strength population in Group D fibres must be due to some subtle difference in processing.

To make strength predictions for long length fibres, the $g(S_m)$ distributions in Fig. 2 must first be extrapolated. It is evident from Fig. 2 that there is some uncertainty in extrapolating these distributions which leads to a predicted range in strengths. The extrapolation of the $g(S_m)$ function was carried out by extending the function from the data range to a strength level corresponding to about 100 MPa, after which the $g(S_m)$ function was extrapolated linearly to 0, 0. The $g(S_m)$ distribution could then be integrated using the trapezoidal rule over the strength range 0 to S_m. The predicted median strength $[\phi(S_m) = 0.5]$ for a given long length was found by determining the strength value where the $\int g(S)\mathrm{d}S$ function was equal to (see Equation 3)

$$\int_0^{S_m} g(S)\mathrm{d}S = \frac{-\ln(1-0.5)}{2rL}. \qquad (5)$$

The results of the long length strength predictions are summarized in Table I. For comparison the predictions based on Weibull unimodal and bimodal strength distributions are also given in Table I [4].

From Table I it is seen that the unimodal Weibull approach does not yield as accurate a failure prediction as the bimodal approach, as would be expected from this obviously bimodal data. The fundamental approach results in a failure prediction range that encompasses the experimentally measured value. These predictions

TABLE I Comparison of predicted and actual fracture strengths at long lengths (L_2) for $\phi = 0.50$

Fibre data	L_2 (m)	Predicted strength at L_2 (MPa)			Actual strength at L_2 (MPa)
		Weibull unimodal	Weibull bimodal	Fundamental	
0.05 m Group B	498	2	203	169–213	179
0.61 m Group B	498	36	240	145–192	179
0.05 m Group D	1100	625	425	104–518	259

illustrate one advantage in using the fundamental approach. Namely, the unqualified use of Weibull statistics for data extrapolation can lead to a false confidence that the strength distribution parameters derived in one strength regime are pertinent to the entire population [11]. This difficulty is largely eliminated with the fundamental approach since the strength range encompassed by the data is clearly indicated (Fig. 2) and the dangers inherent in extrapolating beyond the regime covered by the data become apparent. Also, the Weibull approach places unnecessary restrictions on the functional form of the distribution parameters. Although this problem can be partially counteracted by applying several piece-wise Weibull distributions, this type of data analysis does not represent a realistic mixing of flaw populations [12]. This difficulty is largely eliminated by the fundamental approach.

Acknowledgement

This work was supported by the Office of Naval Research under Contract No. N00014-78-C-0836. The authors are indebted to N. Bandyopadhyay for his help in writing the computer programs and J. Dolan for his assistance in data analysis.

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Received 17 November and accepted 16 December 1980.